Journal of Engineering Physics and Thermophysics, Vol. 67, No. 1-2, 1994

THERMOGRAVITATIONAL CONVECTION IN A SQUARE CELL INVOLVING A PHASE TRANSITION

P. T. Zubkov and K. M. Fedorov

UDC 532.24.001:622.693.4

The processes of freezing and thawing of a liquid in the presence of thermogravitational convection in a square cell with adiabatic vertical and isothermal horizontal walls are considered. The development of convective flow in time depends on the initial conditions, and therefore various scenarios of transition to a stationary solution are possible. The presence of phase transitions in the liquid makes it possible to freeze the transient structure of convective vortices. As a result, depending on the initial conditions, several stationary solutions that differ in both the structure of the convective motion and the magnitude of the heat transfer rate of the medium can exist.

We consider thermogravitational convection in a square cell with adiabatic vertical and isothermal horizontal walls. We will assume that the liquid freezes at $T = T_s$. The system of equations is written in the form

$$\operatorname{div} \mathrm{v} = 0\,, \quad \rho\left[\frac{\partial \mathrm{v}}{\partial t} + (\mathrm{v}\nabla)\,\mathrm{v}\right] = -\nabla p + \mu\Delta \mathrm{v} - g\beta\rho\,(T' - T_0)\,,$$

$$\rho\left[\frac{\partial h}{\partial t} + c\,(\mathrm{v}\nabla)\,T'\right] = \lambda\Delta T'\,, \quad h = cT'\,, \quad T' < T_s\,,$$

$$h = cT' + \kappa\,, \quad T' > T_s\,, \quad h = f\kappa + cT_s\,, \quad T' = T_s\,, \tag{1}$$

i.e., thermogravitational convection is considered using the Boussinesq approximation, and the dependence of enthalpy on temperature is taken to be a piecewise linear function.

We represent system of equations (1) in dimensionless form, for which we introduce the dimensionless coordinates $X = x/l$ and $G = y/l$, where l is the side of the square, and the dimensionless time $\tau = t\nu/l^2$. Here, the characteristic time is taken to be the time of viscous damping $t_0 = l^2/\nu$. In this case system of equations (1) takes the following dimensionless form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0\,, \quad \frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\,,$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\mathrm{Ra}}{\mathrm{Pr}}\,T\,,$$

$$\frac{\partial T}{\partial \tau} + \frac{1}{\mathrm{Ja}}\frac{\partial f}{\partial \tau} + \left(U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)\left(T + \frac{f}{\mathrm{Ja}}\right) = \frac{1}{\mathrm{Pr}}\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right),$$

$$\mathrm{Ra} = \frac{g\beta\,(T_1 - T_0)\,l^3 c\rho}{\nu\lambda}\,, \quad \mathrm{Pr} = \frac{c\nu\rho}{\nu}\,,$$

Institute of the Mechanics of Multiphase Systems. Russian Academy of Sciences, Tyumen'. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 67, Nos. 1-2, pp. 10-13, July-August, 1994. Original article submitted May 11, 1992.

 1062-0125/94/6712-0686$12.50

$$T = \frac{T' - T_0}{T_1 - T_0}, \quad \mathrm{Ja} = \frac{c\rho\,(T_1 - T_0)}{\kappa}. \qquad (2)$$

Thus, the behavior of the system being considered is determined by three dimensionless criterial numbers.

We recall the main results of the theory of natural convection (a detailed review of results and methods of calculation is given in [1]. There is a certain critical value of the Rayleigh number below which the state of rest of the system is stable. Subsequently, with increase in the Rayleigh number, we come to bifurcations of the stationary solution. The stationary solution is observed at values of the dimensionless time $\tau \sim 1$; in problems with phase transitions the stationary solution is obtained at large dimensionless times with a decrease in the Jacob number.

System of equations (2) was solved numerically in a square cell on a grid of 38 $\times$ 38 nodal points. The system of hydrodynamic equations was solved by the SIMPLE method developed by Patankar and Spalding [2]. The heat influx equation was solved by the enthalpy method given in [3]. The numerical model was tested by comparing the results of calculations with the exact one-dimensional solution of the Stefan problem and two-dimensional solutions obtained by other authors on the basis of other numerical models.

To gain an insight into the various regimes of the melting of the liquid, it is worthwhile to consider first the evolution of the solution in the problem of liquid convection without phase transitions. In the calculations the dimensionless parameters had the following values: $\mathrm{Ra} = 5 \cdot 10^4$, $\mathrm{Pr} = 1$, $\mathrm{Ja} \to \infty$. We will consider the evolution of the solution from the state of rest of the liquid. In the first problem suppose that the dimensionless temperature at the initial moment is equal to $T = 1$ over the entire computational domain, except for the upper boundary, where the dimensionless temperature is equal to $T = 0.5$. The side walls are adiabatic. Thus, the first problem is characterized by the following boundary and initial conditions:

$$\tau = 0: \; U = V = 0, \quad T = 1, \quad \tau > 0: \; Y = 0, \quad 0 < X < 1, \quad T = 1,$$

$$Y = 1, \quad 0 < X < 1, \quad T = 0.5, \quad X = 0, \quad X = 1, \quad 0 < Y < 1, \quad \frac{\partial T}{\partial X} = 0.$$

In this case, the liquid in the central region of the cell moves downward initially, forming two symmetric vortices. As the convection develops, one vortex is absorbed by the other, and a vortex is formed that spreads over the entire region of the cell and rotates in a clockwise direction. Just this vortex-type flow is the stationary solution of the problem.

In the second case the convective flow evolves from the state of rest at $T = 0.5$. At the initial instant of time the temperature of the lower wall becomes equal to $T = 1$ and it is then kept constant. In this case a somewhat different evolution of the solution is observed. At the initial instant of time two vortices are also formed, but rotating in the other direction. In this case the stationary convective motion is represented by one vortex but rotating in the opposite direction, i.e., counterclockwise. The solutions obtained show that the symmetry in the convection problems is the condition of total reflection of the flow pattern and not reflection with respect to the symmetry axis. In this connection it should be noted that an artificial solution of the problem in the half-domain leads to a stationary solution consisting of a system of two vortices, whereas the stationary solution of the problem is different (one vortex).

We now consider the process of natural convection complicated by a phase transition. Let the freezing-melting of the liquid occur at $T_s = 0.5$ and the latent heat of melting correspond to the Jacob number $\mathrm{Ja} = 10$. Let us consider three problems of the evolution of the solution to the stationary one for the temperature of the lower wall $T = 1$ and of the upper wall $T = 0$.

In the first case the fluid in the initial state in the cell was in the liquid state at $T = 1$. As with the problem without a phase transition, at first two vortices are formed, with the liquid near the vertical walls moving upward and thus freezing up more intensely at the center of the cell. The phase interface is convex downward and its curvature prevents one vortex from being absorbed by the other. The stationary solution is obtained in the form of two vortices with the phase interface being convex downward (see Fig. 1a).

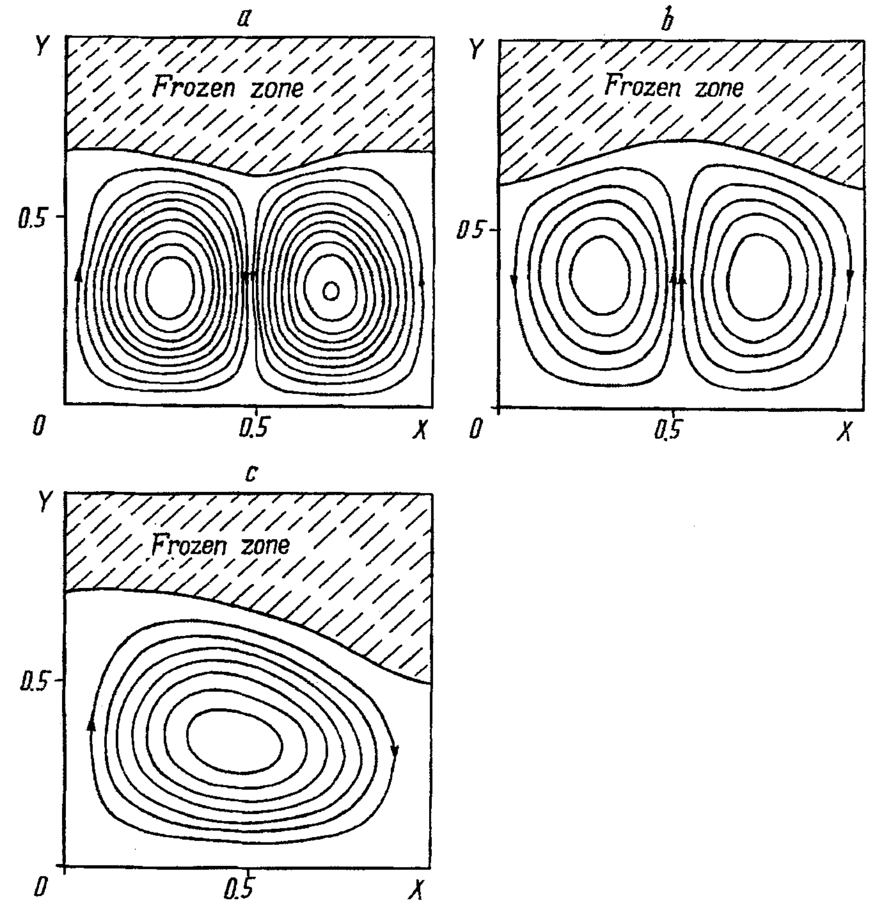

Fig. 1. Streamlines and frozen area in the stationary solution of the problem of the freezing of liquid in a square (a), its thawing (b), and the stepwise freezing of liquid in a square (c).

In the second case the liquid is initially in the solid state at $T = 0$. The melting of the liquid at the temperature of the lower boundary $T(Y = 0) = 1$ occurs as follows. As the melting region appears, convective flow is formed that also consists of a system of two vortices, but the direction of liquid rotation in tem is opposite to the previous case, i.e., the liquid at the center of the cell moves upward. In this case the stationary solution corresponds yo Fig. 1b.

The third solution evolves from the state of rest of the liquid at $T = 1$, but initially on the upper side of the square cell the temperature $T(Y = 1) = 0.5$ is maintained. As soon as one vortex is formed, we decrease the temperature of the upper boundary to $T(Y = 1) = 0$. In this case the liquid freezes with the one-vortex structure of convective flow, and the stationary solution has the form given in Fig. 1c.

A stationary solution with the opposite direction of the rotation of the vortex can be obtained if at first the temperature in the cell is prescribed to be equal to $T = 0.5$ and that of the lower wall to $T(Y = 0) = 1$, i.e., if first the opposite vortex is formed and then the system is allowed to freeze.

The Nusselt number was determined in the three types of stationary solution obtained. It was found that for these three solutions the Nusselt number was equal to 1.546, 1.575, and 1.581, respectively. Thus, the initial conditions influence not only the structure of the vortex flow and the shape of the phase interface but also the integral parameter, i.e., the heat transfer of the cell.

NOTATION

v, velocity vector; p, pressure; c, heat capacity; h, enthalpy; T', temperature; g, acceleration of gravity; f, concentration of nonfrozen liquid in the two-phase region; P, dimensionless pressure; U, dimensionless projection of the velocity onto the OX axis; V, dimensionless projection of the velocity onto the OY axis; T, dimensionless temperature; t, time; l, characteristic scale of the problem; Ra, Rayleign number; Pr, Prandtl number; Ja, Jacob number; κ, latent heat of melting; ∂, kinematic viscosity; ρ, density; ν, thermal conductivity; τ, dimensionless time; x, X, y, Y, dimensional and dimensionless coordinates; Nu, Nusselt number.

REFERENCES

1. E. L. Tarunin, Computational Experiment in Free Convection Problems [in Russian], Irkutsk (1990).
2. S. V. Patankar, Numerical Methods for Solving Problems of Heat Transfer and Liquid Dynamics [in Russian], Moscow (1984).
3. Shamsudar and Sparrow, Trans. ASME, J. Heat Transfer, No. 3, 14-20 (1976).